AIAA JOURNAL
Vol. 35, No. 2, February 1997

# Adaptive Monte Carlo Simulation for Time-Variant Reliability Analysis of Brittle Structures

Sankaran Mahadevan* and Animesh Dey†
*Vanderbilt University, Nashville, Tennessee 37235*

**An efficient simulation-based methodology for estimating the time-dependent reliability of a brittle structural system is presented. In the proposed method, a branch-and-bound method is used to identify the most probable failure sequence of a system. This defines the initial failure domain to start the simulation. As the simulation progresses, the initial failure domain is refined adaptively to incorporate information about the other possible dominant failure sequences, to estimate the true failure probability of the system. The simulation is continued until there is a convergence in the failure probability estimate.**

## I. Introduction

SUBSTANTIAL progress has been made in recent years in the estimation of reliability of structures under uncertain loads. Various structural system configurations have been considered, including series and parallel representations. The structural elements have been modeled as either ductile (i.e., unlimited deformation capacity at the limit load) or brittle (loss of load carrying capacity at the limit load). However, most of these studies have idealized the loading as time-invariant, i.e., as a random variable. The reliability so obtained corresponds to that under one load application, usually some extreme value of the load over a given period of time. In practical situations, most loads vary with time. This variation can be characterized by a random process description. If we consider time dependence, the reliability problem formulation and solution technique will be significantly different from those with time-invariant loads. Furthermore, including the time dependence in the formulation of any reliability problem will help include the effects of degradation of resistance with time. Formulating the problem in the time domain will help address additional issues such as maintenance and rehabilitation of structures, besides giving a more realistic assessment of the reliability estimate.

Several analytical methods based on Markov chain assumptions,[1] the outcrossing approach,[2–5] and FORM/SORM (first-order reliability method/second-order reliability method) methods[6] are available for estimating the time-dependent reliability of structural systems. However, most of these methods are limited by their simplifying assumptions. Furthermore, the computational procedures involved are cumbersome and lengthy. All of these limitations make the analytical methods unappealing and inaccessible to practicing engineers.

Therefore, Monte Carlo simulation seems to be a viable alternative to assess the time-dependent reliability of structural systems. It is easier to implement and is not limited by any simplifying assumptions. Furthermore, realistic conditions, such as strength deterioration, periodic repair of the system, etc., can be incorporated to arrive at a more useful estimate of the time-dependent reliability of a system. However, brute-force Monte Carlo simulation is computationally very expensive for high-reliability problems. Therefore, adaptive methods[7–10] have been developed to increase the efficiency of Monte Carlo simulation. These methods have been proved to be successful for component reliability problems and simple series-type system reliability problems.[11]

An efficient adaptive-importance-sampling-based simulation methodology is developed to estimate the failure probability of brittle systems. A parallel system configuration is assumed and it is shown that the simulation strategy can identify the significant failure sequences efficiently. The application of the proposed methodology is illustrated in detail using a two-bar and a four-bar parallel system. However, these simple structures do not reflect any limitation of the methodology. They are used to facilitate graphical demonstration using probability functions of two random variables. The method is, in fact, the same for more complicated truss and frame structures.[12]

## II. Problem Definition and Assumptions

The reliability problem to be investigated is the probability that a brittle system (elastic brittle) consisting of several members (components) reaches ultimate collapse over a given period of time under the action of one or more time-variant loads. The structure may go through various progressive damage levels before final collapse over a period of time, or there can be sudden failure at any instant. Both possibilities are considered. However, it is assumed that the state of a structure can change only the time of load application. The structure is assumed to respond statically to the load fluctuations and, therefore, dynamic amplification is not considered.

The loads on the structure are modeled as pulse processes in which the load occurrence time and intensity are treated as random variables. The load occurence time is assumed to follow a Poisson process and the load intensity is defined by a Gaussian distribution. The load intensities are assumed to be independent from occurrence to occurrence.

## III. Time-Dependent System Reliability

To illustrate the basic concept of time-dependent reliability analysis, a structural system composed of $m$ components is considered. For the sake of simplicity, the structure is subjected to one load process. The load produces a sequence of $n$ discrete load events over a period of time $(0, t_L]$. The mean occurrence rate of the load is assumed to be $\nu$. For the present, the initial resistances of the structural components are assumed to be deterministic and equal to $\boldsymbol{r} = \{r_1, \ldots, r_m\}$. The strength of the $i$th component deteriorates with time according to $r_i(t) = r_i \times g_i(t)$, where $r_i(t)$ is the strength of the $i$th component at time $t$ and $g_i(t)$ is a deterministic degrading function of time. The intensity of the load at each occurrence is assumed to be $\boldsymbol{s} = \{s_1, \ldots, s_n\}$. The loads induce a structural action $c_i s_j$ in the $i$th component at the $j$th occurrence, for a parallel system failure will occur only when the strongest of all the components has failed. Conversely, for the whole system to survive, the strongest component must survive for all $n$ occurrences of the load. Therefore, the reliability of the system can be expressed as

$$\begin{aligned} L_S &= P\left[\max_{i=1}^{m} r_i g_i(t_1) > c_i s_1 \cap \cdots \cap \max_{i=1}^{m} r_i g_i(t_n) > c_i s_n\right] \\ &= P\left[\max_{i=1}^{m} \frac{r_i g_i(t_1)}{c_i} > s_1 \cap \cdots \cap \max_{i=1}^{m} \frac{r_i g_i(t_n)}{c_i} > s_n\right] \\ &= \prod^{n} F_S\left[\max_{i=1}^{m} \frac{r_i g_i(t_j)}{c_i}\right] \end{aligned} \tag{1}$$

Received July 22, 1996; revision received Oct. 1, 1996; accepted for publication Oct. 27, 1996; also published in *AIAA Journal on Disc*, Volume 2, Number 2. 

*Associate Professor, Department of Civil and Environmental Engineering. Member AIAA.

†Graduate Student, Department of Civil and Environmental Engineering.

where $F_S(\ )$ is the cumulative distribution function of the load intensity. The time of load occurrences $(t_j)$ in the above expression have been considered as deterministic. But for a Poisson pulse process, the interarrival time of the load is exponentially distributed. Therefore, considering the randomness in the interarrival times of the load occurrences, Eq. (1) is rewritten as[13]

$$L_S = \left\{ \int_0^{t_L} F_S\left[ \max_{i=1}^{m} \frac{r_i g_i(t)}{c_i} \right] \frac{1}{t_L}\, \mathrm{d}t \right\}^n \tag{2}$$

Because the load is described by a Poisson pulse process, the probability that there will be $n$ occurrences in the time period $(0, t_L]$ is

$$P[N(t_L) = n] = \frac{(\nu t_L)^n \times \exp(-\nu t_L)}{n!}$$

Using the theorem of total probability, and considering all possible values of $n$, the reliability of the system then can be written as

$$L_S = \sum_{n=0}^{\infty} \left\{ \int_0^{t_L} F_S\left[ \max_{i=1}^{m} \frac{r_i g_i(t)}{c_i} \right] \frac{1}{t_L}\, \mathrm{d}t \right\}^n \times \frac{(\nu t_L)^n \exp(-\nu t_L)}{n!}$$

$$= \exp\left( -\nu \cdot \left\{ t_L - \int_0^{t_L} F_S\left[ \max_{i}^{m} \frac{r_i g_i(t)}{c_i} \right] \mathrm{d}t \right\} \right)$$

The failure probability of the system can be represented as

$$P_f(t_L \mid \boldsymbol{R} = \boldsymbol{r}) = 1 - L_S$$
$$= 1 - \exp\left( -\nu \cdot \left\{ t_L - \int_0^{t_L} F_S\left[ \max_{i}^{m} \frac{r_i g_i(t)}{c_i} \right] \mathrm{d}t \right\} \right)$$

As a final step, if $f_{\boldsymbol{R}}(\boldsymbol{r})$ be the joint probability density of the initial strength of the components, the failure probability of the system can be rewritten as

$$p_f(t_L) = \int_0^{\infty} P_f(t_L \mid \boldsymbol{R} = \boldsymbol{r}) f_{\boldsymbol{R}}(\boldsymbol{r})\, \mathrm{d}r \tag{3}$$

$$= \int_0^{\infty} \{1 - \exp[\ ]\} f_{\boldsymbol{R}}(\boldsymbol{r})\, \mathrm{d}r \tag{4}$$

It is clear that the outer integral in Eq. (4) is a multidimensional integral (over the whole domain of the component resistance variables). Even for a simple two-bar parallel system, computation of the integral can be very cumbersome. Therefore, to estimate the time-dependent failure probability of a system, an efficient sampling technique is developed that computes the outer multidimensional integral; the inner single dimensional integral over the time domain is determined by analytical means. The details of the proposed method are discussed in the following sections.

## IV. Adaptive Importance Sampling

In many practical situations, for structures with a high reliability, brute-force Monte Carlo simulation can be a very inefficient (both time and cost) method to estimate their failure probability because of the large number of simulations required. Importance sampling is a method used to make the sampling technique more efficient. The concept of importance sampling is explained in this section.

The basic formula to compute the probability of failure is

$$p_f = \int_{g(x) \le 0} f_X(x)\, \mathrm{d}x$$

where $f_X(x)$ is the joint probability density function of the input random variables and $g(x) = 0$ is the limit state equation. Dividing the set of random variables into two mutually exclusive subsets, $\tilde{X}$ and $\check{X}$, the failure probability can be expressed as[14]

$$p_f = \int_{\tilde{x}} P[\, g(\check{x}, \tilde{x}) \le 0 \mid \tilde{X} = \tilde{x}] f_{\tilde{X}}(\tilde{x})\, \mathrm{d}\tilde{x}$$

$$= \int_{\tilde{x}} p_{f \mid \tilde{X} = \tilde{x}}\, f_{\tilde{X}}(\tilde{x})\, \mathrm{d}\tilde{x}$$

$$= E[p_{f \mid \tilde{X} = \tilde{x}}] \tag{5}$$

Therefore, if a Monte Carlo simulation is carried out with $N$ samples, the failure probability estimate will be

$$p_f \approx \frac{1}{N} \sum_{i}^{N} p_{f \mid \tilde{X} = \tilde{x}^{(i)}}$$

where the sample outcomes $\tilde{x}^i$ are generated from the probability density $f_{\tilde{X}}(\tilde{x})$ and $p_{f \mid \tilde{X} = \tilde{x}^i}$ is the conditional failure probability given $\tilde{X} = \tilde{x}^i$. Equation (5) can be rewritten as

$$p_f = \int_{\tilde{x}} \frac{p_{f \mid \tilde{X} = \tilde{x}}\, f_{\tilde{X}}(\tilde{x})}{h_{\tilde{X}}(\tilde{x})} h_{\tilde{X}}(\tilde{x})\, \mathrm{d}\tilde{x} \tag{6}$$

$$= E\left[ \frac{p_{f \mid \tilde{X} = \tilde{x}}\, f_{\tilde{X}}(\tilde{x})}{h_{\tilde{X}}(\tilde{x})} \right] \tag{7}$$

where $h_{\tilde{X}}(\tilde{x})$ is the new sampling density function that focuses the sampling on the more important regions in the failure domain and thus helps in faster convergence to the true failure probability.

Comparing Eqs. (5) and (3), it is observed that they are of very similar nature. The vector of resistance variables $\boldsymbol{R}$ corresponds to the set $\tilde{X}$, and the load variable $S$ corresponds to the set $\check{X}$. Therefore, a conditional importance sampling approach can be employed to estimate the time-dependent failure probability of a system.

However, selecting the right sampling function, $h_{\tilde{X}}(\tilde{x})$, is not an easy task. Some studies[9,15] have shown that if the sampling function is chosen inappropriately, the results may be quite erroneous. Karamchandani and Cornell[14] have suggested that the correct position of the sampling function can be determined iteratively by shifting the mean value of the sampling function to the point of maximum $p_{f \mid \tilde{X} = \tilde{x}} f_{\tilde{X}}(\tilde{x})$ in Eq. (6). The assumption in adopting such a scheme is that there is only one region where the integrand in Eq. (6) is maximum. This assumption need not be valid always. As will be shown with the help of examples later, the integrand in Eq. (3), $L_{S \mid R = r} f_R(r)$, can have more than one region of importance. Therefore, to estimate the true failure probability accurately, the sampling function must identify all of the important regions. Several approaches[8,11,16] have been proposed for identifying the correct form and position of the sampling density function.

The method that is used here is based on the technique developed by Karamchandani et al.[9] In this method, the initial sampling density function is chosen to have the same form and variance as the original density function but centered around the most probable point (MPP) in the failure domain. Once several samples have been obtained in the failure domain, a multimodal sampling density function is constructed that emphasizes multiple points in the failure domain, each in proportion to the true probability density at the point. However, not all sample points are emphasized; only one representative point from a cluster of points is chosen. The representative points are separated by a distance greater than the "cluster radius" $d_0$ (see Fig. 1). Usually the value of $d_0$ is taken to be the average distance between

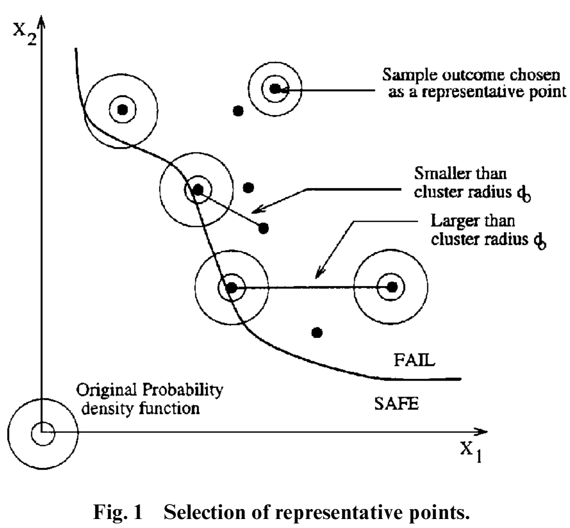


**Fig. 1 Selection of representative points.**

the MPP and the sampling points. The multimodal sampling density to generate the $i$th sample point is

$$h^i_X(\tilde{x}) = \sum_{j=1}^{k} \hat{\omega}^j_i f^{(j)}_X(\tilde{x}) \tag{8}$$

where $\hat{\omega}^j_i$ = importance attached to the $j$th sampling point,

$$p_{f\,|\,\tilde{X}=\hat{x}^{(j)}} f_{\tilde{X}}\left[\hat{x}^{(j)}\right] \Big/ \left\{ \sum_{r=1}^{k} p_{f\,|\,\tilde{X}=\hat{x}^{(j)}} f_{\tilde{X}}\left[\hat{x}^{(r)}\right] \right\}$$

$f_X(x)$ = original density function
$f^{(j)}_X(x)$ = original density function with the mean shifted to $\hat{x}^{(j)}$
$\hat{x}^{(1)} \ldots \hat{x}^{(k)}$ = representative points

The representative points are identified as follows. Let $S_i$ be the set of all previously identified sample points in the failure domain including the MPP. A cluster radius $d_0$ is selected. The point with the largest probability density in $S_j$ is selected and is called $\hat{x}^{(1)}$. In $S_i$, all points within a radius $d_0$ of $\hat{x}^{(1)}$ are eliminated. Among the remaining points in $S_i$, the point with the largest probability density is selected and called $\hat{x}^{(2)}$. All points within a radius $d_0$ of $\hat{x}^{(2)}$ are eliminated. This process is repeated until there are no more points left in $S_i$.

After $i$ sample points, the estimated failure probability is given by

$$p_f = \frac{1}{i} \sum_{j=1}^{i} \frac{p_{f\,|\,\tilde{X}=x^{(j)}} f_{\tilde{X}}\left[x^{(j)}\right]}{h^j_{\tilde{X}}\left[x^{(j)}\right]} \tag{9}$$

## V. Proposed Methodology

Generally speaking, when the members of a redundant structure behave in a brittle manner under some loading, the reliability analysis becomes difficult. When a member reaches its limit, it loses its load-carrying capacity and there is load redistribution among the surviving members. A statically indeterminate system goes through many levels of progressive damage (failure) before final collapse, under a load process. Also, depending upon the load path, a structure may go through different failure sequences before final collapse.[1,17] Therefore, even for a simple structure such as a four-bar truss, there can be many failure sequences. The total failure probability of the structure will be the probability of union of all failure sequences. To determine the individual failure probabilities of all sequences is a cumbersome process and might defeat the whole purpose of using even a computationally simple method such as Monte Carlo simulation. Therefore, to reduce the amount of computation, a branch-and-bound technique[18] is used to identify the significant failure sequences of a given system. Among the significant failure sequences of a structure, only the most probable sequence will contribute the maximum to the total failure probability of a system. The most probable failure sequence is a sequence with the highest probability of failure in its first node. Therefore, to further reduce the amount of computation, only the most probable failure sequence is chosen for the initial simulation.

It was explained in Sec. IV that the initial sampling in adaptive importance sampling technique starts around the MPP for a single limit state problem. Drawing analogy from the technique, the most probable failure sequence will define the initial failure domain for the adaptive importance sampling technique. Once the initial sampling is done, a set of representative points are chosen that emphasize the region of the most probable failure sequence. With this initial set of representative points, further sampling is carried out with increased variance to identify other important regions in the sampling domain corresponding to the other dominant failure sequences of the system. After each sampling, the set of representative points is modified to reflect the true nature of the failure domain. In this manner, the whole sampling domain is mapped to estimate the failure probability accurately. Sampling is continued until the failure probability converges to an accepted level of accuracy.

## VI. Numerical Examples

### A. Two-Bar Parallel System

The two-bar parallel system shown in Fig. 2 is used to clarify the concepts discussed in the preceding sections. The statistical parameters of the input random variables are described in Table 1. It is assumed that bar 1 takes 30% of the load $S(t)$ and bar 2 carries the remaining 70% of the load. If one of the bars fails, the surviving bar carries the full load until failure. There are only two possible failure sequences for this system: bar 1 followed by bar 2 (1-2) and bar 2 followed by bar 1 (2-1).

The structure was analyzed for failure probability for up to 50 years. But to understand the concepts of the proposed method, we investigate the simulation results for 10 years in more detail.

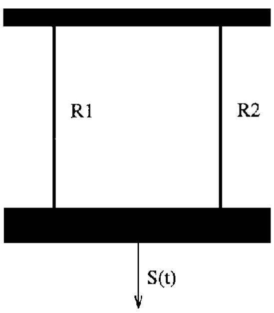


**Fig. 2 Two-bar parallel system.**

**Table 1 Statistics of variables for a two-bar parallel system**

| Random variables | $\mu$ | $\sigma$ | $\nu$ |
|---|---|---|---|
| R1 | 1.5 | 0.5 | |
| R2 | 4.0 | 0.5 | |
| $S(t)$ | 3.5 | 0.3 | 1/yr |

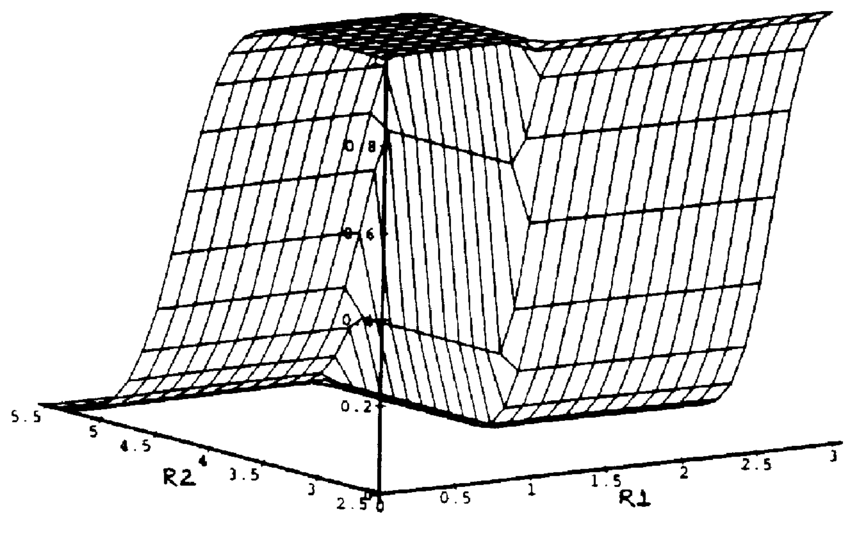


**Fig. 3 Conditional failure probability for different values of R1 and R2.**

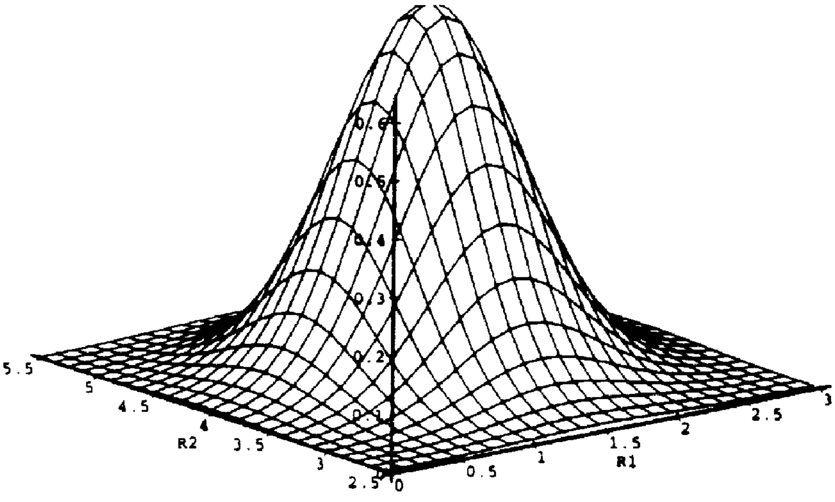


Fig. 4 Joint probability density function of R1 and R2.

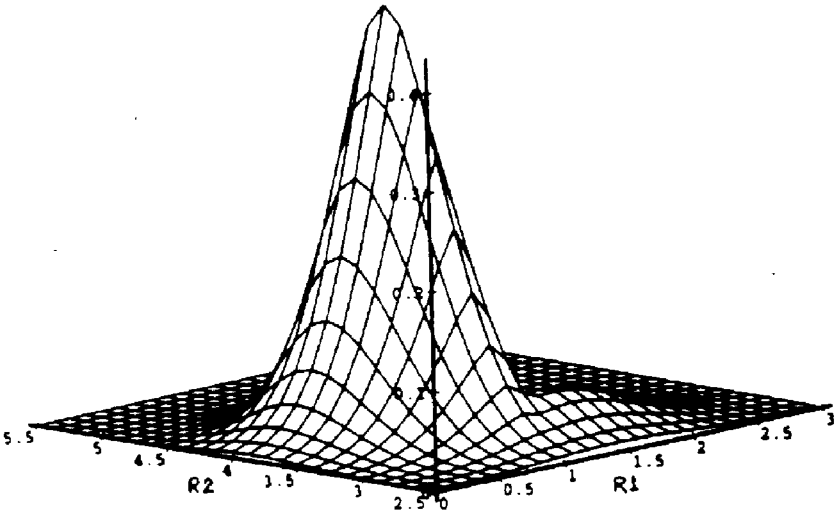


Fig. 5 Plot of the integrand in Eq. (3).

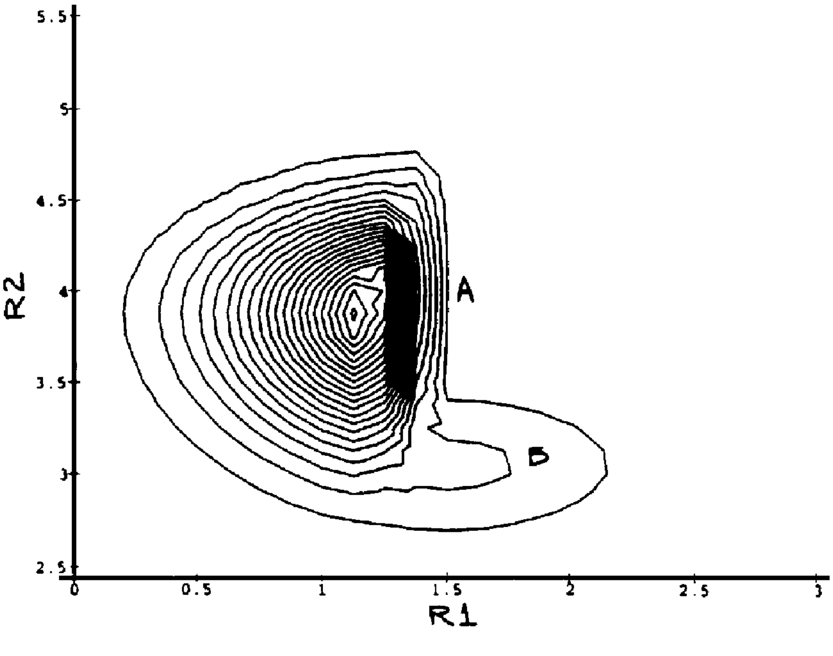


Fig. 6 Contour plot of the integrand in Eq. (3).

Table 2 Simulation results for the two-bar parallel system

| Years | Monte Carlo $10^5$ simulations | Adaptive sampling (simulations) |
|---|---|---|
| 10 | 0.1342 | 0.1258 (58) |
| 20 | 0.1665 | 0.1672 (45) |
| 30 | 0.1876 | 0.1892 (79) |
| 40 | 0.2004 | 0.2010 (59) |
| 50 | 0.2089 | 0.2070 (62) |

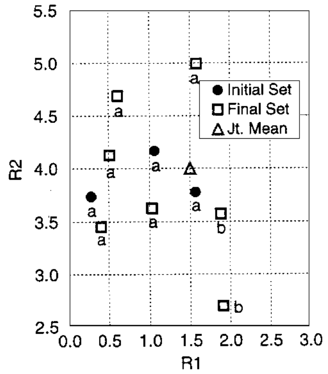


**Fig. 7 Representative points.**

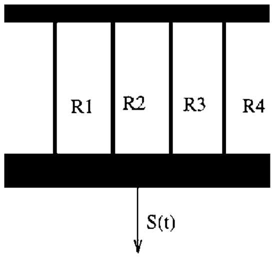


**Fig. 8 Four-bar parallel system.**

Between the two failure sequences, sequence 1-2 is more probable. Figure 3 shows the plot of the failure probability for different values of R1 and R2. Figure 4 is the joint probability density of the resistance variables, $f_R(r)$. Figures 5 and 6 show the plot and contour plot, respectively, of the integrand in Eq. (4), which is simply the product of the expressions plotted in Figs. 3 and 4. The volume under the surface in Fig. 5 is the failure probability of the system. Observing the contour plot (Fig. 6), it is clear that there are two regions (A and B) in the sampling domain where the integrand is important. Region A is more important compared to region B.

As discussed in Sec. V, the initial simulation was concentrated around the most probable failure sequence (1-2). After the initial simulation, further simulation was carried out to identify the other important regions of the sampling domain. Figure 7 shows a plot of the representative points as the simulation progressed. It is clear from the plot that the initial set of representative points was concentrated on the most probable sequence. But by the end of the simulation, the set of representative points mapped the integrand quite well according to the importance of the region. The points marked "a" emphasize the region A (sequence 1-2), and the points marked "b" emphasize the region B (sequence 2-1). Because region A is more important than region B (see Fig. 5), there are more points in region A than in region B (see Fig. 7). It is observed from Table 2 that the proposed method is in good agreement with the basic Monte Carlo simulation results and also that convergence to the true solution is very fast. The structure is deliberately designed to have high failure probabilities to facilitate the verification of the proposed method with a small number of simulations.

### B. Four-Bar Parallel System

Figure 8 shows the four-bar parallel system considered for this example. The statistical parameters of the input variables are shown in Table 3. It is assumed that there is equal distribution of the load among the four bars. Among the many possible failure sequences, the significant failure sequences, identified by the branch-and-bound method, are 1-2-4-3, 2-1-4-3, and 1-4-2-3 (Ref. 1). Among the three significant failure sequences, sequence 1-2-4-3 is the most probable. Therefore, as discussed before, the initial simulation is centered around this sequence and gradually the failure domain was adapted to include the other possible sequences. The simulation results (not shown here) showed that during the course of sampling, representative points corresponding to sequences 2-1-4-3 and 1-4-2-3 were picked up. It is observed form Table 4 that there is good agreement between the proposed method and the basic Monte Carlo simulation results and also that convergence to the true solution is very fast.

Table 3 Statistics of variables for a four-bar parallel system

| Random variables | $\mu$ | $\sigma$ | $\nu$ |
|---|---|---|---|
| R1 | 1.5 | 0.45 | |
| R2 | 2.1 | 0.62 | |
| R3 | 6.0 | 1.2 | |
| R4 | 3.0 | 0.3 | |
| $S(t)$ | 1.0 | 1.44 | 1/yr |

Table 4 Simulation results for the four-bar parallel system

| Years | Monte Carlo $10^6$ simulations | Adaptive sampling (simulations) |
|---|---|---|
| 10 | 0.000766 | 0.000779 (45) |
| 20 | 0.00146 | 0.00154 (45) |
| 30 | 0.0028 | 0.00245 (72) |
| 40 | 0.00304 | 0.00298 (77) |
| 50 | 0.00377 | 0.00373 (105) |

## VII. Conclusion

Most of the analytical methods available for estimating the time-dependent reliability of structural systems tend to be complicated and unattractive. Moreover, sometimes the application of such methods is limited by their simplifying assumptions. In this paper, an efficient importance-sampling-based methodology was developed for time-dependent failure probability estimation of parallel brittle systems. Unlike many analytical methods, no simplifying assumptions about load overlapping need to be made by this method. Compared to the basic Monte Carlo simulation, convergence to the true solution is faster by the proposed method. The proposed method was demonstrated with the help of two simple examples.

## Acknowledgment

This research was partly supported by a grant from Vanderbilt University Research Council.

## References

[1] Mahadevan, S., and Xiao, Q., "Time-Variant Reliability of Brittle Structural Systems," *Journal of Engineering Mechanics*, Vol. 121, No. 3, 1995, pp. 414–423.

[2] Rice, S. O., "Mathematical Analysis of Random Noise," *Bell System Technical Journal*, Vol. 23, 1944, pp. 282–332; Vol. 24, 1945, pp. 46–156; reprinted in Wax, N., *Selected Papers on Noise and Stochastic Processes*, Dover, New York, 1954.

[3] Melchers, R. E., "Sampling Techniques for Time-Variant Reliability Problems," *Probabilistic Mechanics and Structural and Geotechnical Reliability, Proceedings of the Sixth Specialty Conference* (Denver, CO), edited by Y. K. Lin, American Society of Civil Engineers, New York, 1992, pp. 100–103.

[4] Chan H. Y., "System Reliability Analysis with Time-Dependent Loads and Resistances," Ph.D. Dissertation, Dept. of Civil Engineering, Univ. of Newcastle, New South Wales, Australia, 1991.

[5] Rackwitz, R., "Reliability of Systems Under Renewal Pulse Loading," *Journal of Engineering Mechanics*, Vol. 111, No. 9, 1985, pp. 1175–1184.

[6] Guers, F., Dolinski, K., and Rackwitz, R., "Probability of Failure of Brittle Redundant Structural Systems in Time," *Structural Safety*, Vol. 5, 1988, pp. 169–185.

[7] Harbitz, A., "An Efficient Sampling Method for Probability of Failure Calculation," *Structural Safety*, Vol. 3, 1986, pp. 109–115.

[8] Bucher, C. G., "Adaptive Sampling—An Iterative Fast Monte Carlo Procedure," *Structural Safety*, Vol. 5, 1988, pp. 119–126.

[9]Karamchandani, A., Bjerager, P., and Cornell, C. A., "Adaptive Importance Sampling," *Proceedings of the 5th International Conference on Structural Safety and Reliability* (San Francisco, CA), edited by A. H.-S. Ang, M. Shinozuka, and G. I. Schuëller, American Society of Civil Engineers, New York, 1989, pp. 855–862.

[10]Fu, G., "Variance Reduction by Truncated Multimodal Importance Sampling," *Structural Safety*, Vol. 13, 1994, pp. 267–283.

[11]Mori, Y., and Ellingwood, B. R., "Time-Dependent System Reliability Analysis by Adaptive Importance Sampling," *Structural Safety*, Vol. 12, 1993, pp. 59–73.

[12]Dey, A., and Mahadevan, S., "Time-Dependent Reliability Analysis of Redundant Brittle Systems," *Probabilistic Mechanics and Structural Reliability, Proceedings of the Seventh Specialty Conference* (Worcester, MA), edited by D. M. Frangopol and M. D. Grigoriu, American Society of Civil Engineers, New York, 1996, pp. 700–703.

[13]Taylor, H. M., and Karlin, S., *An Introduction to Stochastic Modeling*, Academic, Orlando, FL, 1984.

[14]Karamchandani, A., and Cornell, C. A., "Adaptive Hybrid Conditional Expectation Approaches for Reliability Estimation," *Structural Safety*, Vol. 11, 1991, pp. 59–74.

[15]Ibrahim, Y., "Observations on Applications of Importance Sampling in Structural Reliability Analysis," *Structural Safety*, Vol. 9, 1991, pp. 269–281.

[16]Melchers, R. E., "Search-Based Importance Sampling," *Structural Safety*, Vol. 9, 1991, pp. 117–128.

[17]Wen, Y. K., and Chen, H.-C., "System Reliability Under Time Varying Loads: II," *Journal of Engineering Mechanics*, Vol. 115, No. 4, 1989, pp. 824–839.

[18]Xiao, Q., and Mahadevan, S., "Fast Failure Mode Identification for Structural System Reliability," *Structural Safety*, Vol. 13, 1994, pp. 207–226.